Chapter 1 to 7 End Test

/50 Marks

1. a. Find the exact solution of the equation $2e^{6x} - 3e^{3x} - 5 = 0$.

[3]

$2e^{6x} - 3e^{3x} - 5 = 0$

let $e^{3x} = u$

$2u^2 - 3u - 5 = 0$

$2u^2 + 2u - 5u - 5 = 0$

$2u(u+1) - 5(u+1) = 0$

$(2u-5)(u+1) = 0$

$u = 5/2$ or -1 (reject)

$u = 5/2$

$e^{3x} = 5/2$

$3x = \ln(5/2)$

$x = \dfrac{\ln(5/2)}{3}$

b. Solve the following simultaneous equation

$$e^{4x-7} \div e^{5x+7y} = \frac{1}{e^2}$$

$$xy + 18 = 0$$

[5]

$e^{4x-7} \div e^{5x+7y} = e^{-2}$

$e^{4x-7-(5x+7y)} = e^{-2}$

$e^{4x-7-5x-7y} = e^{-2}$

$-x-7-7y = -2$

$-x-7y = 5$

$x = -7y-5$

$xy+18 = 0$

$(-7y-5)(y)+18 = 0$

$-7y^2-5y+18 = 0$

$7y^2+5y-18 = 0$

$(7y-9)(y+2) = 0$

$y = 9/7$ or -2

$\therefore x = -14$ or 9

2. Variables x and y are such that when e^{4y} is plotted against x, a straight line of gradient $\frac{2}{5}$, passing through (10, 2), is obtained.

a. Find y in terms of x. [3]

$e^{4y} = mx + c$

$e^{4y} = \frac{2}{5}x + c$

$2 = \frac{2}{5}(10) + c$

$c = -2$

$e^{4y} = \frac{2}{5}x - 2$

$4y = \ln\left(\frac{2}{5}x - 2\right)$

$y = \frac{\ln\left(\frac{2}{5}x - 2\right)}{4}$

b. Find the value of y when x = 45, giving your answer in the form $\ln p$. [2]

$y = \frac{\ln\left(\frac{2}{5}(45) - 2\right)}{4}$

$= \frac{\ln(16)}{4}$

$= \frac{1}{4}\ln(16) = \ln(16)^{1/4} = \ln 2$

$p = 2$

c. Find the values of x for which y can be defined. [1]

$\frac{2}{5}x - 2 > 0$

$\frac{2}{5}x > 2$

$x > 5$

3. Solve the equation $4|7x - 3| - 5 = 9$.

[3]

$4|7x-3| = 14$

$|7x-3| = \frac{14}{4}$

$7x-3 = \frac{14}{4}$ or $-14/4$

$7x-3 = 14/4$

$7x = 13/2$

$x = 13/14$

$7x-3 = -14/4$

$7x = -1/2$

$x = -\frac{1}{14}$

4. **DO NOT USE A CALCULATOR IN THIS QUESTION.**

Variables *x* and *y* are related by the equation $y = kx^2$. When $x = 1 + \sqrt{2}$, $y = 1 - \sqrt{2}$. Find the value of *k*, giving your answer in the form $a + b\sqrt{c}$, where *a*, *b* and *c* are integers.

[4]

$y = kx^2$

$1-\sqrt{2} = k(1+\sqrt{2})^2$

$= k(1+2\sqrt{2}+2)$

$1-\sqrt{2} = k(3+2\sqrt{2})$

$k = \frac{1-\sqrt{2}}{3+2\sqrt{2}}$

$= \frac{1-\sqrt{2}\,(3-2\sqrt{2})}{(3+2\sqrt{2})(3-2\sqrt{2})} = \frac{3-2\sqrt{2}-3\sqrt{2}+4}{9-8}$

$= 7-5\sqrt{2}$

5. The points *A*, *B* and *C* have coordinates (2, 6), (6, 1) and (*p*, *q*) respectively. Given that *B* is the mid-point of *AC*, find the equation of the line that passes through *C* and is perpendicular to *AB*. Give your answer in the form *ax*+*by* = *c*, where *a*, *b* and *c* are integers. [5]

$B \to (6, 1)$

$\hookrightarrow \left(\frac{2+p}{2}, \frac{6+q}{2}\right)$

$p = 10, q = -4$

$C \to (10, -4)$

$m_{AB} = \frac{1-6}{6-2} = -5/4$

Normal $= \frac{-1}{-5/4} = 4/5$

$y - -4 = 4/5(x - 10)$

$y + 4 = 4/5x - 8$

$y = 4/5x - 12$

$5y = 4x - 60$

$4x - 5y = 60$

6. a. Find the range of values of *x* satisfying the inequality $(5x - 1)(6 - x) < 0$. [2]

$x = 1/5$ or 6 $\longrightarrow$

$\therefore$ $x < 1/5$ or $x > 6$

b. Show that the equation $(2k + 1)x^2 - 4kx + 2k - 1 = 0$, where $k \neq -\frac{1}{2}$, has distinct, real roots. [3]

$b^2 - 4ac$.

$(-4k)^2 - 4(2k+1)(2k-1)$.

$16k^2 - 4(4k^2 - 1)$

$16k - 16k^2 + 4$

$4 > 0$

$\therefore$ real roots

7. (a)(i) Write down the set of values of *x* for which lg (5*x* - 3) exists.

[1]

$5x-3>0$ $\quad x>3/5$

$5x>3$

(ii) Solve the equation lg (5*x* - 3) = 1.

[1]

$5x-3=10$

$5x=13$

$x=13/5$

(b) It is given that $log_y x = 4 + \frac{1}{2}log_y 64 + log_y 162$, where $y > 0$. Find an expression for y in terms of *x*. Simplify your answer.

[5]

$\log_y x = 4+\log_y 8 + \log_y 162$

$\log_y x = 4+\log_y 1296$

$\log_y x = \log_y y^4 + \log_y 1296$

$\log_y x = \log_y 1296y^4$

$x = 1296y^4$

$\sqrt[4]{x} = 6y$

$y = \frac{\sqrt[4]{x}}{6}$

8. The polynomial $p(x) = 6x^3 + ax^2 + 6x + b$, where *a* and *b* are integers, is divisible by $2x - 1$. When p(*x*) is divided by $x - 2$, the remainder is 120.
(a) Find the values of *a* and *b*.

[4]

$p(\tfrac{1}{2}) = 0$, $p(2) = 120$

$0 = 6(\tfrac{1}{2})^3 + a(\tfrac{1}{2})^2 + 6(\tfrac{1}{2}) + b$

$0 = \tfrac{3}{4} + \tfrac{a}{4} + 3 + b$

$0 = \tfrac{15}{4} + \tfrac{a}{4} + b$

$0 = 15 + a + 4b$

$120 = 6(2)^3 + a(2)^2 + 6(2) + b$

$120 = 48 + 4a + 12 + b$

$60 = 4a + b$

$b = 60 - 4a$

$15 + a + 4(60 - 4a) = 0$

$15 + a + 240 - 16a = 0$

$-15a = -255$

$a = 17 \therefore b = 60 - 4(17)$

$= -8$

(b) Hence write down the remainder when p(*x*) is divided by *x*.

[1]

-8

9. Find the possible values of *k* for which the equation $kx^2 + (k+5)x - 4 = 0$ has real roots.

[5]

$b^2 - 4ac > 0$

$(k+5)^2 - 4(k)(-4) = 0$

$k^2 + 10k + 25 + 16k = 0$

$k^2 + 26k + 25 = 0$

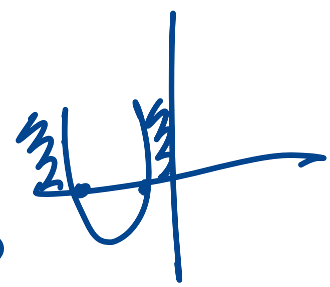
$k^2 + k + 25k + 25 = 0$

$k(k+1) + 25(k+1) = 0$

$(k+1)(k+25) = 0$

$k = -1$ or -25

$k > -1$ or $k < -25$



10. Solve the equation $\dfrac{625^{\frac{x^3-1}{2}}}{125^{x^3}} = 5.$

[2]

$\dfrac{(5^4)^{\frac{x^3-1}{2}}}{(5^3)^{x^3}} = 5^1$

$\dfrac{5^{2x^3-2}}{5^{3x^3}} = 5^1$

$2x^3 - 2 - 3x^3 = 1$

$-x^3 = 3$

$x^3 = -3$

$x = \sqrt[3]{-3}$

$= -1.44$